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Tooth flank approximation with root point iteration – potentials and limits in gear metrology

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1. Introduction and state of the art

The production of gears demands high-precision engineering. Due to the narrow geometric tolerances, the golden rule of metrology (measuring a feature with an uncertainty less than 10 % of its tolerance) can often not be reached during quality inspection. Thus, the international standard defines 20–30 % as acceptable [1], and toothing qualities grade 3 or better cannot be proven on an industrial level. In order to increase the producible quality of gears, the geometric assessment has to become more reliable [2]. Numerous developments are in progress for the uncertainty reduction in gear inspection, dealing with calibration [3,4] and data acquisition by means of optical measurement principles that work with point [5], line [6] or area-based [7] measurement approaches. This article, however, focuses on the investigation of the uncertainty contributions from the data evaluation.

The evaluation of geometric gear data is divided into two fields, the determination of unknown gear parameters as well as the quality inspection of gears with a known nominal geometry. The estimation of unknown gear parameters requires a partitioning of the measuring data into different geometric areas, e. g., the effective involute, tip or root reliefs [8]. Automatic partitioning approaches can be clustered into the following categories: edge detection, region growing, attribute clustering and hybrid methods [9]. While edge-based methods are sensitive to noisy data [10] and will not succeed in identifying smooth transition borders [11], region-based methods tend to over- or under-segmentation [10]. None of those methods seems to be suitable for an accurate partitioning of gear data and was ever applied for it. In contrast, attribute-clustering methods can be robust [10] and, if a priori knowledge is provided by a geometric model, the achievable precision can be increased in order to fulfil the high demands of gear technology. One implementation of attribute clustering is the holistic approximation (HA), which is a model-based method that combines the automated partitioning with the gear parameter approximation [12,13]. While the HA was introduced for the 3D geometry evaluation of micro features [14], the recent enhancement by a root point iteration [15] in principle enables the estimation of gear parameters of not only unmodified but also modified flanks. However, the application of the HA to automatically determine unknown geometric parameters of modified involute flanks is pending.

The other field of data evaluation, the quality inspection of involute gears, is a well-standardized process according to ISO 1328–1 [8]. The standard evaluation of profile and helix deviations is a two-step approximation process in two cross-sections in 2D. For gears with unmodified flanks, the orthogonal distances between the measurement points and the geometric gear model, which are required for the approximation, can be calculated directly [16]. For modified flanks, however, an iterative root point calculation is necessary to determine the orthogonal distances, which is currently missing in the standard procedure. Instead, the root points on the unmodified flank are used. Currently it is not clear, how this inaccuracy depicted in Fig. 1 influences the uncertainty of the evaluated deviation parameters, like profile or helix slope deviations [8]. The enhanced HA offers the possibility to correctly determine the root points even on modified flanks, but it was not yet applied for this gear inspection task.

Therefore, the aim of the article is to prove the applicability of the HA for the estimation of unknown toothing parameters and the inspection of modified gears, and it is assessed whether the integrated root point iteration enables more precise results or an accelerated gear

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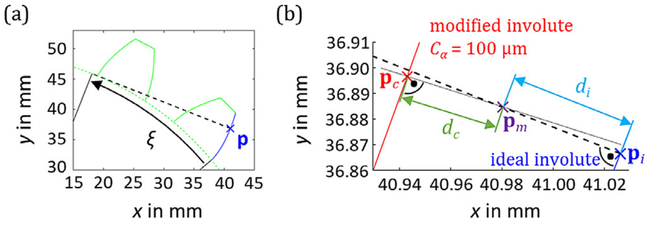

Fig. 1. (a) Geometry of an involute gear with the point p on an involute flank and the corresponding roll angle ξ, defined by the surface normal, which is a tangent to the base circle (green dotted line). The black dashed line is the roll length, which is the product of the roll angle ξ and the base circle radius r_b. (b) Detail view around a measuring point $\mathbf{p}_m$ with the different orthogonal distances d_i and d_c to the root points $\mathbf{p}_i$ and $\mathbf{p}_c$ on an ideal involute (unmodified) and a crowned involute (modified), respectively. In case of crowned flanks, the standard data evaluation uses the distance d_i to the unmodified flank instead of calculating the distance d_c to the modified flank.

quality inspection with a decreasing point density. For this purpose, the HA method applied for the geometric evaluation of a gear profile of a crowned involute with tip relief is described in Section 2. The application is focused on external helical gears, as this type of gear has the greatest relevance for actual practical applications. The results are presented and discussed in Section 3, divided into the estimation of unknown gear parameters and gear inspection. Section 4 finally summarizes the findings.

2. Method

The holistic approximation uses a parametric geometric model in combination with a least squares approach. The following sub-sections introduce the geometric model of the measurement object, the HA approach as well as the root point iteration.

2.1. Geometric gear model

As a gear with a modified tooth flank, the geometric combination of a crowned involute (see Fig. 2a) with a tip relief (see Fig. 2b) within the transverse section is considered. The resulting profile with its shape parameters $\mathbf{a}_s = [r_b, C_\alpha, r_{b,r}]$ and position parameters $\mathbf{a}_p = [\psi_b, \psi_{b,r}, r_r]$ are presented in Fig. 2c. Here, r_b and $r_{b,r}$ are the base circle radii of the involute and of the tip relief, respectively, C_α is the crowning of the involute, ψ_b and $\psi_{b,r}$ are the polar angles of the involute and the tip relief, respectively, and r_r is the starting radius of the tip relief. This model assumes that the workpiece coordinate system is estimated in advance by measuring reference datum elements, as usual in the standard inspection procedure.

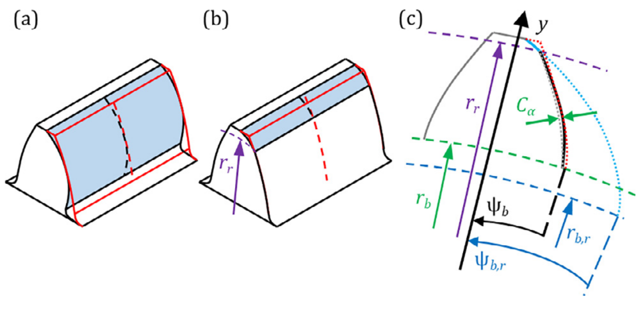

Fig. 2. Flank modifications (the red lines indicate the unmodified flank, the red and black dashed lines are the profile lines of the unmodified and the modified flank, respectively) and the geometric model: (a) Crowning in profile direction. (b) Tip relief on an involute flank, starting at radius r_r. (c) Geometric model with feature parameters: base circle radii r_b and $r_{b,r}$ of the involute and the tip relief, respectively, crowning amplitude C_α, polar angles (base thickness half angles) ψ_b and $\psi_{b,r}$ of the involute and the tip relief, respectively, as well as the starting radius of the tip relief r_r. The red and grey dotted lines represent the unmodified involute, and the blue dotted line is the involute of the tip relief.

The HA is implemented from the point of view of geometric data processing, using shape and position parameters to define the approximating elements. Therefore, the respective base circle radii are evaluated as shape parameters for the involutes instead of the modules or helix angles. Note that it is common for gear inspection to use specially defined deviation parameters to describe geometric deviations [8], which are related to the shape parameters of the geometric elements. As an example, a tip relief is usually identified by its pressure angle $\alpha_{t,r}$, but can also be assessed by the base circle radius $r_{b,r}$ according to the geometrical relation

$$r_{b,r} = 0.5 \cdot \frac{m_n}{\cos(\beta)} \cdot Z \cdot \cos(\alpha_{t,r}), \tag{1}$$

with the normal module m_n, the helix angle β and the number of teeth Z.

2.2. Holistic approximation (HA)

To obtain the best estimates of the shape and position parameters $\mathbf{a}_s$, $\mathbf{a}_p$, the approach is to minimize the summed squares of the $k = 1, \ldots, N_j$ orthogonal point distances $d_{j,k}$ for the $j = 1, \ldots, N_E$ integral geometric elements, weighted by individual factors $w_{j,k}$:

$$\min_{\mathbf{a}_p, \mathbf{a}_s} \left[\sum_{j=1}^{N_E} \sum_{k=1}^{N_j} w_{j,k} \cdot \{ d_{j,k}(\mathbf{a}_p, \mathbf{a}_s) \}^2 \right]. \tag{2}$$

Note that during the optimization of the free geometric parameters $\mathbf{a}_p$ and $\mathbf{a}_s$, the assignment of the measuring points to the geometric elements is also adapted, i. e. the numbers of points N_j in general vary during the iterative optimization.

For the modified gear described in Section 2.1, the $N_E = 2$ integral elements are the involute and the tip relief with N_1 and N_2 measuring points, respectively. Although the weighting factor is a free design parameter of the HA to optimize the result, an equal weighting factor $w_{j,k} = 1$ is initially used in the subsequent demonstration. As decision rule for the assignment, the polar radius of the measuring point r_m is compared to the position parameter r_r in the actual iteration step:

- $r_m < r_r$: The measuring point is assigned to the involute.
- $r_m > r_r$: The measuring point is assigned to the tip relief.

As a result, the HA combines parameter estimation and partitioning in a single holistic optimization routine.

2.3. Root point iteration

The HA minimizes the orthogonal distances of the measuring points to the approximating elements, which is referred to as geometric fitting or best fit approximation. In contrast to an unmodified involute, where the orthogonal distance from a measuring point can be calculated directly [16], the distance calculation for the considered modified involute requires an iterative root point estimation. The orthogonal distance

$$d = \| \mathbf{p}_m - \mathbf{p}_c \|_2 \tag{3}$$

is defined as the Euclidean norm of the measuring point $\mathbf{p}_m = \{x_m, y_m, z_m\}$ and the root point $\mathbf{p}_c = \{x_c, y_c, z_c\}$, cf. Fig. 1. It can be calculated with the constraint $g(\mathbf{p}_c = f(\mathbf{a}_p, \mathbf{a}_s)) = 0$, which means the root point has to be on the nominal geometry described by the function f. Accordingly, the orthogonal distances are calculated by solving the constrained minimization task

$$\min_{\mathbf{p}_c} \left[\| \mathbf{p}_m - \mathbf{p}_c \|_2 + \lambda \cdot g(\mathbf{p}_c = f(\mathbf{a}_p, \mathbf{a}_s)) \right] \tag{4}$$

with the Lagrange multiplier λ by using the iterative Newton method. This root point iteration is a cascaded minimization within the approximation task. Thus, in each approximation step, the parameters $\mathbf{a}_p$ and $\mathbf{a}_s$ are constant during the root point iteration. Finally, the correct root point eliminates the Lagrange term in Eq. (4) so that it equals Eq. (3).

3. Results

The HA is verified and validated by means of simulations and experiments, respectively, for both the determination of unknown gear parameters and the gear inspection with known nominal gear parameters. For validation purposes, profile lines are measured on a helical gear (number of teeth $Z = 40$, module $m_n = 2.0$ mm, pressure angle $\alpha_t = 18°$, helix angle $\beta = -30.75°$) with modified flanks (crowning $C_\alpha = 5.0$ µm, tip relief $\alpha_{t,r} = 23°$) using a coordinate measuring machine (CMM) with 1.0 mm diameter ruby balls, see Fig. 3.

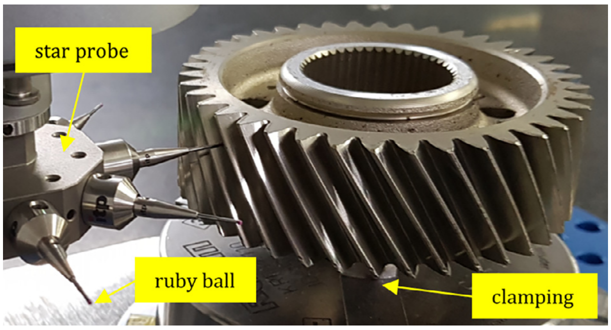

Fig. 3. Tactile measurements on a helical gear with modified flanks using a star probe with 1.0 mm diameter ruby balls.

3.1. Estimation of unknown gear parameters

The fundamental geometry of an involute toothing is defined by four parameters: module, pressure and helix angle as well as number of teeth. The reverse engineering of unknown parameters requires the partitioning of different geometric areas on the flanks, and the HA offers an automated optimal partitioning. However, there is no reproducible possibility to generate a reference for real measurement data of modified gears to compare with, as no software is available which offers an adequate partitioning. Thus, the partitioning results are validated qualitatively by Fig. 4, showing a comprehensible assignment of each point to the involute or the tip relief region with orthogonal distances to the estimated gear geometry smaller than 1 µm.

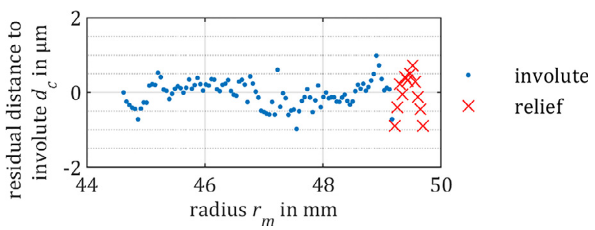

Fig. 4. Result of the automatic partitioning of the holistic approximation for real measurement data with the residual orthogonal distances d_c to the crowned involute and the tip relief, respectively. The standard deviation is 0.34 µm for the involute residuals and 0.53 µm for the relief residuals.

Furthermore, the HA is verified using simulated measuring points with an equally distributed measurement uncertainty in profile normal direction within the range [-$a_e/2$ ... $a_e/2$]. Here, 100 points per profile are simulated for a similar gear geometry with normal module $m_n = 5.0$ mm, $Z = 21$ teeth, pressure angle $\alpha_t = 20°$, pressure angle $\alpha_{t,r} = 30°$ of the tip relief and crowning $C_\alpha = 20$ µm.

In addition to the measurement uncertainty, a random profile slope deviation is included in each simulated gear profile measurement by varying the base circle radius with a normal distribution with 5 µm standard deviation. The simulation is repeated 100 times for different noise levels $a_e/2$, and the evaluated deviations δr_b to the nominal base circle radius $r_{b,0} = 49.3339$ mm are presented in Fig. 5.

The mean base circle radius deviations (black crosses in Fig. 5) are below 0.2 µm with standard uncertainties in the same order of magnitude. In order to test for systematic deviations, an analysis of variances (AnOVa) is performed. Firstly, the homogeneity of variances is analyzed by means of a Levene test [17], which shows that equal variances cannot be assumed. Therefore, a Welch-AnOVa is applied to investigate systematic influences. The result of the Welch test is $F_{rb}(4;495) = 0.67 < 2.39 = F_{crit}(0.05;4;495)$. As a result, with a probability of error of 5 %, the HA results of the estimated base circle radius contain no systematic deviations.

To characterize the random deviations, the standard deviation of the base circle radius is normalized to the individual noise ranges a_e (red circles in Fig. 5). As a result, it amounts to 70–90 % of the noise range of the data acquisition. Indeed, the increase of the standard deviation of the estimated base circle radius with an increasing noise level $a_e/2$ agrees with the result of Levene's test concerning the variance inhomogeneity. Hence, the uncertainty of the HA is mainly scaled by the noise of the data acquisition, but note that the random deviations are also determined by the number of measuring points.

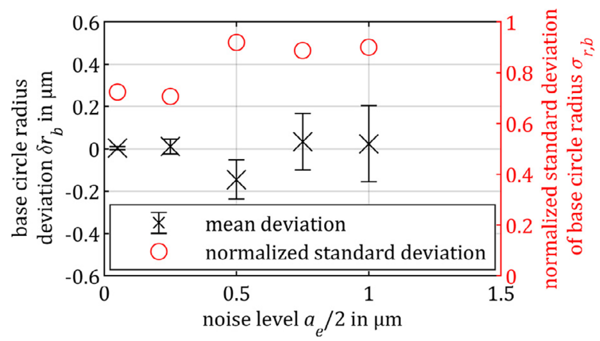

Fig. 5. Mean base circle radius deviations δr_b of 100 repeated simulations over the level $a_e/2$ of equally distributed noise as position measurement noise (the error bars represent the standard error) together with the normalized standard deviation $\sigma_{r,b}$ of the approximated base circle radii, normalized by the range a_e of the position measurement noise.

As a result, the base circle radius of a crowned flank with tip relief is reliably estimated, which demonstrates the capability of the HA for an automated estimation of unknown geometric parameters of modified involute gears. With the base circle radius as a fundamental shape parameter, the remaining gear parameters can be estimated according to Eq. (1) with additional measurements, e. g., in helix direction.

3.2. Gear inspection with known nominal gear parameters

The two-step approximation is the standard evaluation procedure for a modified profile, but uses an incorrect root point for calculating the orthogonal distance, see Fig. 1. Therefore, it is studied whether the HA is an alternative inspection approach and if the iterative root point determination, that is integrated in the HA, leads to more precise results.

The profile slope deviation for a randomly chosen tooth is evaluated as reference with the commercial software QUINDOS to $f_{H\alpha,\mathrm{ref}} = -0.8$ µm within the diameters $D_{Cf} = 89.14$ mm and $D_{Fa} = 97.36$ mm, and the crowning is determined to the reference value $C_{\alpha,\mathrm{ref}} = 5.0$ µm. The actual base circle radius

$$r_\mathrm{b} = r_\mathrm{b,nom} \cdot \left(1 + \frac{f_{H\alpha}}{L_\alpha}\right) \tag{5}$$

follows from the nominal base circle radius $r_{b,\mathrm{nom}}$, the profile slope deviation $f_{H\alpha}$ and the evaluation length L_α. This linear relation is valid for unmodified involutes and can be used to determine the reference value $r_{b,\mathrm{ref}} = 43.5337$ mm, as the standard two-step evaluation approximates an unmodified flank.

The surface coordinates measured by the CMM are also processed by the HA. Note that the evaluation boundaries are set to the same values as in the reference evaluation for the sake of comparability, i. e. the automatic partitioning capability needs to be disabled for the comparison. The HA calculates the crowning amplitude to $C_{\alpha,\mathrm{HA}} = 5.1$ µm. In order to compare the base circle radii, the crowning of the approximating element must be set to zero and this degree of freedom is locked in a second approximation run in the HA. As a result, the HA calculates $r_{b,\mathrm{HA}} = (43.5339 \pm 0.0008)$ mm. The uncertainty was estimated for this example based on the estimations in Section 3.1 (Fig. 5) and a probing error of $a_e = 0.9$ µm ($k_p = 1$). The differences to the reference values are

0.1 µm (C_α) and 0.2 µm (r_b), respectively, and are not significant with respect to the measurement uncertainty. This means, on the one hand, that the effect of using a wrong root point in the standard evaluation is negligible. On the other hand, the HA can be considered as validated for the inspection of modified gears, since the HA algorithm is able to reach the same accuracy as the standard evaluation.

3.3. *Optimized gear inspection*

In order to analyze whether the HA is able to evaluate more precise results in combination with a decreasing point density, the set of 111 measuring points is evaluated with both the HA and the commercial software in the same evaluation range, and the point density is stepwise reduced by a factor f_r. The resulting base circle radius deviations Δr_b to the reference value with no point reduction are presented in Fig. 6.

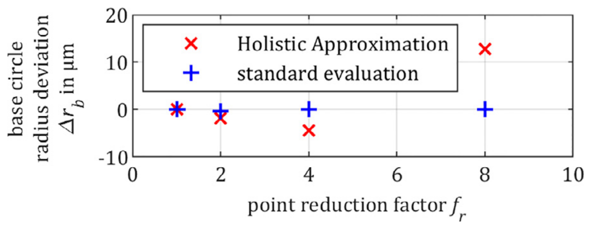

Fig. 6. Base circle radius deviations Δr_b caused by evaluating a reduced number of points (point reduction factor f_r) compared for standard evaluation and holistic approximation.)

As a result, there is no significant difference in the effort, but the standard evaluation procedure is more robust with respect to a decreasing point density or increasing factor f_r, respectively. Using the HA, a reduction of the point density by a factor of 8 leads to deviations of the base circle radius of more than 10 µm with respect to the result without reducing the point density. The reason is that the more accurate distance calculation within the HA requires additional degrees of freedom for the cascaded root point approximation, cf. Eq. (4), and this multidimensional optimization is found to be more sensitive to a reduced signal-to-noise ratio or point density, respectively. However, the required maximum lateral point distance for the HA with root point iteration of 0.1 µm is in perfect agreement with the ISO 1328−1 [8].

4. Summary

The extension of the HA by a root point iteration is validated for the determination of unknown gear parameters for crowned involutes as well as for gear inspection with nominal parameters. As a result, the HA provides a reliable and automated estimation of unknown gear parameters. For this task, no systematic deviations are observed, and standard deviations of 70 to 90 % of the noise level of the input data are reached for involutes acquired with 100 points.

The standard gear inspection procedure with known nominal gear parameters contains two evaluation steps, which imply errors in calculating the orthogonal distances of the measuring points to a crowned involute. The studied alternative approach is approximating the measuring points directly by a crowned involute, e. g., by the capabilities of the extended HA. Whereas for the standard point density both approaches deliver similar results with deviations below 0.2 µm, a detailed comparison revealed that the standard two-step approximation is less sensitive to a decreasing point density of noisy data. Although using the correct root point, the HA does not significantly decrease the measurement uncertainty. But, on the other hand, the HA is fast and its implicit optimal partitioning enables automated evaluations and is able to deliver results with minimized uncertainty if the evaluation range is not specified or unknown.

As a result, the HA is demonstrated for the first time to be applicable for gear measurement tasks with modified flanks. As outlook, a 3D implementation of the HA for gear measurement tasks seems promising, since 3D inspection was already demonstrated for micro drawing dies [14]. Thus, in the future, the HA will also be capable for the calculation of areal gear deviation parameters.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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